Analysis of Time Series:

Moving Average Method:

Ex 1 / LU 2010

Taking 3 yearly moving average calculate trend value:

Years	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
Production	21	22	23	25	24	22	25	26	27	26.

Years	Production	3 Yearly Totals	3 Yearly Average	5 Yearly Totals	5 Yearly Average
1995	21	-	-	-	-
1996	22	66	22.00	-	-
1997	23	70	23.33	115	23.0
1998	25	72	24.00	116	23.2
1999	24	71	23.67	118	23.8
2000	22	71	23.67	122	24.4
2001	25	73	24.33	124	24.8
2002	26	78	26.00	126	25.2
2003	27	79	26.33	-	-
2004	26.	-	-	-	-

(i) Starting Year = $\frac{N+1}{2}$ where N is yearly Average to be calculated. Here $S = \frac{3+1}{2} = 2^{nd}$ Year

(ii) Add 3-three years to get 3 yearly Totals & divide Totals by 3 to get Average

(iii) The average yields the trend of the Series; which in this question is an increasing trend.

(i) Incase of Yearly Moving Average $S = \frac{N+1}{2}$ or $S = \frac{5+1}{2} = 6/2 = 3^{rd}$ Year.

(ii) Add 5-5 years progressively to get 5 yearly Totals & divide totals by 5 to get Average. Which also indicates trend of the Series.

(iii) In 5 yearly Moving Average we find a clear moving average.

Ex 2 Calculate trend by 4 yearly moving average method:

Years	Output.	4 Yearly Moving Average		
		4 Yrly Totals	Average	Centring.
2012	4	-	-	-
2013	8	-	-	-
		40	10	
2014	16			→ 11.5
		52	13	
2015	12			→ 14.5
		64	16	
2016	16			→ 16.5
		68	17	
2017	20	-	-	- -
		-	-	
2018	20	-	-	- -

(i) $S = \frac{N+1}{2} = \frac{4+1}{2} = 2.5^{th}$ Year.

(ii) After getting 4 yearly Totals & then averages.

(iii) We go for centring which is centre of averages.

(iv) Centring gives the trend of the series.

(v) In the given series trend is an increasing one.

Ex 3 Computation of Trend by Method of Least Squares:

Fit a Secular Trend by least square method to following data & Calculate expected value for 2021.

Years (X)	Output (in lakhs) (Y)	x (X−M)	$x.Y$	x^2	Trend: $Y = 5 + 0.5(x)$
2014	4	−2	−8	4	$5+0.5(-2) = 4.0$
2015	6	−1	−6	1	$5+0.5(-1) = 4.5$
2016	3	0	−	−	$5+0.5(0) = 5.0$
2017	5	1	5	1	$5+0.5(1) = 5.5$
2018	7	2	14	4	$5+0.5(2) = 6.0$

$\Sigma Y = 25$ $\Sigma x = 00$ $\Sigma xY = 5$ $\Sigma x^2 = 10$

Steps: 1. Put Years as X & Output as Y. & make ΣY

2. Take middle Year & find x which is X−middle Year (M)
3. Find Σx which would be zero
4. Find $x.Y$ (product of x & Y) & $\Sigma x.Y$.
5. Calculate x^2 & find Σx^2.
6. Use least square formula to get value of a & b.

$$\Sigma Y = Na + b\Sigma x$$
$$\Sigma xY = a\Sigma x + b\Sigma x^2$$

7. The fit a straight line Trend by the straight line formula $Y = a + bx$.

In the giving question:

$\Sigma Y = Na + b\Sigma x$ or $25 = 5a + b.0$ or $a = 25/5 = 5$

$\Sigma xY = a\Sigma x + b\Sigma x^2$ or $5 = 0 + 10b$ or $b = 5/10 = 0.5$.

Trend line = $Y = 5 + 0.5(x)$

Expected Value for 2021

$x = X - M$ or $x = 2021 - 2016 = 5$

Value $(Y) = 5 + 0.5(5)$

$= 5 + 2.5$ or $Y = 7.5$ lakhs.

Ex 4

LU 1998 Calculate trend by using method of Least Squares :

Year X	Value Y	x X-M	xY	x^2	$Y = 60 + (-3)x$.
1986	75	-6	-450	36	or $Y = 60 + (-3)(-6) = 78$
1989	67	-3	-201	9	or $Y = 60 + (-3)(-3) = 69$
1991	68	-1	-68	1	or $Y = 60 + (-3)(-1) = 63$
1992	65	0	00	-	or $Y = 60 + (-3)(0) = 60$
1994	50	2	100	4	or $Y = 60 + (-3)(2) = 54$
1995	54	3	162	9	or $Y = 60 + (-3)(3) = 51$
1997	41	5	205	25	or $Y = 60 + (-3)(5) = 45$

$\Sigma Y = 420$ $\Sigma x = 0$ $\Sigma xY = -252$ $\Sigma x^2 = 84$

Using method of least squares :

$\Sigma Y = Na + b\Sigma x$ or $420 = 7a + b.0$ or $a = 60$

$\Sigma xY = a\Sigma x + b\Sigma x^2$ or $-252 = 0. + 84b$ or $b = -3$. rate of increase is -3.

a gives initial value & b indicates rate of increase per year.

Ex 5 / LU 2011 Calculate trend by method of least squares: & estimate value for the year 2000

Year X	Value Y	x X-M	xY	x^2	$y = a + bx$ or $Y = 60 + 5(x)$
1995	45	-2	-90	4	$60 + 5(-2) = 50$
1996	56	-1	-56	1	$60 + 5(-1) = 55$
1997	78	0	0	-	$60 + 5(0) = 60$
1998	46	1	46	1	$60 + 5(1) = 65$
1999	75	2	150	4	$60 + 5(2) = 70$

$\Sigma Y = 300$ $\Sigma x = 00$ ΣxY 50 $\Sigma x^2 = 10$

Using least square eqns:

$\Sigma Y = Na + b\Sigma x$ or $300 = 5a + 0$ or $a = 60$

$\Sigma xY = a\Sigma x + b\Sigma x^2$ or $50 = 0 + 10b$ or $b = 50/10 = 5$

Value for the Year 2000

$x = 2000 - 1997 = 3$ & $Y = 60 + 5(3) = 75$.

Ex 6 / LU 2013.

Year X	Sales Y	x X-M	xY	x^2	$y = a + bx$ or $y = 120 + 10(x)$
2007	90	-2	-180	4	$y = 120 + 10(-2) = 100$
2008	112	-1	-112	1	$= 120 + 10(-1) = 110$
2009	156	0	-	-	$= 120 + 10(0) = 120$
2010	92	1	92	1	$= 120 + 10(1) = 130$
2011	150	2	300	4	$= 120 + 10(2) = 140$

$\Sigma Y = 600$ $\Sigma x = 00$ $\Sigma xY = 100$ $\Sigma x^2 = 10$

$\Sigma Y = Na + b\Sigma x$ or $600 = 5a + 0$ or $a = 120$

$\Sigma xY = a\Sigma x + b\Sigma x^2$ or $100 = 0 + 10b$ or $b = 10$

Estimated value for 2012 is: $120 + 10(3)$ or 150.

Ex 7

From the following data calculate trend & estimate the value for 2025

Year X	Output Y	x (X−M) M(X−2017.5)	$x' = 2x$	$x'Y$	x'^2	$y = 10 + 0.3(x')$
2016	8	−1.5	−3	−24	9	10+0.3(−3) = 9.1
2017	12	−0.5	−1	−12	1	10+0.3(−1) = 9.7
2018	9	+0.5	+1	9	1	10+0.3(1) = 10.3
2019	11	+1.5	+3	33	9	10+0.3(3) = 10.9
	$\Sigma Y = 40$	$\Sigma x = 00$	$\Sigma x' = 00$	$\Sigma x'Y = 6$	$\Sigma x'^2 = 20$	

$\Sigma Y = Na + b\Sigma x$ or $40 = 4a + 0$ or $a = \frac{40}{4} = 10$

$\Sigma x'Y = a\Sigma x' + b\Sigma x'^2$ or $6 = 0 + 20b$ or $b = \frac{6}{20} = 0.3$

Trend value for 2025 is: $x = X - M$ or $2025 - 2017.5 = 7.5$ $x' = 7.5 \times 2 = 15$

or $Y = 10 + 0.3(15)$ or $Y = 10 + 4.5$ or $Y = 14.5$.

Ec 8

Calculate trend on the basis of method of least squares:

Years X	income Y	x X-M	$x' = 2x$ 2(X-M)	$x'Y$	x'^2	Trend $Y = a + bx'$ or $Y = 83.33 + 8.2857x'$
2016	60	-2.5	-5	-300	25	83.33 + 8.2857 (-5) = 41.9
2017	80	-1.5	-3	-240	9	83.33 + 8.2857(-3) = 58.47
2018	100	-0.5	-1	-100	1	83.33 + 8.2857(-1) = 75.04
2019	90	+0.5	1	90	1	83.33 + 8.2857(1) = 91.62
2020	110	+1.5	3	330	9	83.33 + 8.2857(3) = 108.19
2021	160	+2.5	5	800	25	83.33 + 8.2857(5) = 124.7 = 124.76
	$\Sigma Y = 500$		$\Sigma x' = 00$	$\Sigma x'Y = 580$	$\Sigma x'^2 = 70$	

Middle year in this case is 2018.5 $\quad M = \frac{N+1}{2} = \frac{6+1}{2} = 3.5^{th}$ Year

Choose x' as $2x$ to obtain full nos & remove fractions.

Using Eqns of method of Least Squares: Here x is replaced by x'

$\Sigma Y = Na + b\Sigma x'$ or $500 = 6a + 0$ or $a = \frac{500}{6}$ = or $a = 83.33$

$\Sigma x'Y = a\Sigma x + b\Sigma x'^2$ or $580 = 0 + 70b$ or $b = \frac{580}{70}$ or $b = 8.2857$.

Ex 8 LU 2012

Fit a Second degree parabola to the following data:

(Note: Second degree parabola Eqn is $y = a + bx + cx^2$)

Year X	Value Y	x X-M	xY	x^2	x^2Y	x^3	x^4	Fitting Second degree parabola: $y = a + bx + cx^2$ or $Y = 5.97 + 1.6(x) + 0.714(x^2)$
1991	5	-2	-10	4	20	-8	16	$5.97 + 1.6(-2) + 0.7(4) = 5.57$
1992	7	-1	-7	1	7	-1	1	$5.97 + 1.6(-1) + 0.7(1) = 5.07$
1993	4	-	-	-	-	-	-	$5.97 + 1.6(0) + 0.7(0) = 5.97$
1994	9	1	9	1	9	1	1	$5.97 + 1.6(1) + 0.7(1) = 8.27$
1995	12	2	24	4	48	8	16	$5.97 + 1.6(2) + 0.7(4) = 11.97$

$\Sigma Y = 37$ $\Sigma x = 00$ $\Sigma xY = 16$ $\Sigma x^2 = 10$ $\Sigma x^2Y = 84$ $\Sigma x^3 = 00$ $\Sigma x^4 = 34$

Eqns for least squares:

$\Sigma Y = Na + b\Sigma x + c\Sigma x^2$ or $37 = 5a + 0 + 10c$ — ①

$\Sigma xY = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$ or $16 = 0 + 10b + 0$ or $b = 1.6$ — ②

$\Sigma x^2Y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$ or $84 = 10a + 0 + 34c$ — ③

Considering eqn ① & ③ & Multiplying eqn ① by 2 we get.

$10a + 20c = 74$ — ④

$10a + 34c = 84$ — ⑤

Substracting we have: $-14c = -10$ or $c = \frac{+10}{+14} = 0.714$

Putting the value of c in eqn ④ we have:

$10a + 20 \times 0.714 = 74$ or or $10a = 74 - 14.28$ or $a = 5.97$

Ex 9
LU 2011

Year X	Sales Y	x (X-M)	xY	x^2	x^2Y	x^3	x^4	$Y = 12.314 - 0.6x - 0.857x^2$
2004	10	-2	-20	4	+40	-8	16	$12.314 - 0.6(-2) - 0.857(4) = 10.086$
2005	12	-1	-12	1	+12	-1	1	$12.314 - 0.6(-1) - 0.857(1) = 12.057$
2006	13	0	-	-	-	-	-	$12.314 - 0.6(0) - 0.857(0) = 12.314$
2007	10	1	10	1	10	1	1	$12.314 - 0.6(1) - 0.857(1) = 10.857$
2008	8	2	16	4	32	8	16	$12.314 - 0.6(2) - 0.857(4) = 7.686$

$\Sigma Y = 53$ $\Sigma x = 00$ $\Sigma xY = -6$ $\Sigma x^2 = 10$ $\Sigma x^2Y = 94$ $\Sigma x^3 = 00$ $\Sigma x^4 = 34$

Applying Eqn of least squares of second degree parabola we have:

$\Sigma Y = Na + b\Sigma x + c\Sigma x^2$ or $53 = 5a + 0 + 10c$ — ①

$\Sigma xY = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$ or $-6 = 0 + 10b + 0$ or $b = -0.6$ — ②

$\Sigma x^2Y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$ or $94 = 10a + 0 + 34c$ — ③

Taking Eqn ① & ③ & Multiplying eqn ① by 2 we have:

$10a + 20c = 106$ — ④

$10a + 34c = 94$ — ⑤

Substracting we have or $-14c = 12$ or $c = -0.857$

Substituting value of c in eqn ④ we have:

$10a + 20(-0.857) = 106$

or $10a = 17.14 + 106$ or $10a = 123.14$ or $a = 12.314$.

In the given question:

$Y = 12.314 - 0.6x - 0.857x^2$ or $Y = 12.314 - 0.6x - 0.857x^2$

Ex 10
LU 2003

Fit a parabolic curve of second degree to the following data.

Years (X)	Sales (in '000) (Y)	x (X-M)	$x.Y$	x^2	x^2Y	x^3	x^4	$Y = 6.14 + 1.2(x) + 0.43(x^2)$
1991	5	-2	-10	4	20	-8	16	$6.14 + 1.2(-2) + 0.43(4)$
1992	7	-1	-7	1	7	-1	1	$6.14 + 1.2(-1) + 0.43(1)$
1993	4	0	0	-	-	-	-	$6.14 + 1.2(0) + 0.43(0)$
1994	9	1	9	1	9	1	1	$6.14 + 1.2(1) + 0.43(1)$
1995	10	2	20	4	40	8	16	$6.14 + 1.2(2) + 0.43(4)$

$\Sigma Y = 35$ $\Sigma x = 00$ $\Sigma xY = 12$ $\Sigma x^2 = 10$ $\Sigma x^2Y = 76$ $\Sigma x^3 = 00$ $\Sigma x^4 = 34$

Applying Eqns of Least Squares we get:

$\Sigma Y = Na + b\Sigma x + c\Sigma x^2$ or $35 = 5a + 0 + 10c$ or $5a + 10c = 35$ — ①

$\Sigma xY = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$ or $12 = 0 + 10b + 0$ or $b = 1.2$ — ②

$\Sigma x^2Y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$ or $76 = 10a + 0 + 34c$ or $10a + 34c = 76$. — ③

Taking eqn ① & ③ & multiply eqn ① by 2 we get.

$10a + 20c = 70$ — ④

$10a + 34c = 76$ — ⑤

Substracting: $-14c = -6$ or $c = 6/14 = 0.43$

Putting value of c in eqn ④ we have.

$10a + 20(0.43) = 70$ or $10a = 70 - 8.6$ or $a = 6.14$.

Fitting values in eqn of Parabola we have:

$Y = 6.14 + 1.2(x) + 0.43(x^2)$

Ex 1 LU 1994, 2006

Seasonal Variation Indicies

(43)

Year	I Qtr	II Quarter	III Quarter	IV Quarter
1981	72	68	80	70
1982	76	70	82	74
1983	74	66	84	80
1984	84	86	90	80
1985	76	74	84	78
1986	82	86	88	92
1987	94	98	96	92

Soln:

Quarterly Totals	558	548	604	566
Quarterly Average $\frac{\text{Qtrly Totals}}{\text{No of Years}}$	79.71	78.29	86.29	80.857
General Average (Av. of Averages)		$\frac{325.147}{4} = 81.287$		
Seasonal Indicies: $\left(\frac{\text{Quarterly Av}}{\text{General Av}} \times 100\right)$	98.05	96.31	106.15	99.47

Ex 2
LV 2008

Find out Seasonal Variation Indices from the following data :

Year	I Quarter	II Quarter	III Quarter	IV Quarter
2001	30	40	36	34
2002	34	52	50	44
2003	40	58	54	48
2004	54	76	68	65
2005	80	92	86	82
Ofs-Seasonal Totals	238	318	294	273
Quarterly Average	47.6	63.6	58.8	54.6
General Average :	—	56.15	—	
Seasonal Indicies	84.77	113.27	104.72	97.24

Ex 3

L.U. 1998

Year	I Quarter	II Quarter	III Quarter	IV Quarter
1990	24	28	32	40
1991	38	40	48	52
1992	56	60	40	42
1993	46	48	50	56
1994	60	68	72	74
Quarterly Total	224	244	242	264
Quarterly Average	44.8	48.8	48.4	52.8
General Average	—	48.7	—	
Seasonal Indices	91.99	100.2	99.38	108.42